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ABSTRACT

The "Preparing Elementary Teachers to Teach Mathematics (PETTM) Project" (1988-1991) was a cooperative university/school effort to improve the teaching of mathematics by elementary teachers with its primary focus on improving the university training of preservice teachers (PSTs) in mathematics. This document, volume II of the project's final report, describes the "Content Component" of the program. An introductory section provides the background and rationale for the project and a description of its four components. An overview of the content component is given followed by a description of M118, a required finite mathematics course for elementary education majors. The next section describes T104, a newly developed mathematics course for PSTs with problem solving and exploration at its core. Publications and presentations related to the content component and a description of related instructional products are provided. (MDH)

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Preparing Elementary Teachers to Teach Mathematics:

A Problem-solving Approach

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Final Report

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PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS: A PROBLEM-SOLVING APPROACH

OVERVIEW OF THE ENTIRE PROJECT

The *Preparing Elementary Teachers to Teach Mathematics Project* (1988-1991) was a cooperative university/school effort to improve the teaching of mathematics by elementary teachers. The primary focus of the project was the improvement of the university training of preservice teachers (PSTs) in mathematics. In the late-1980s, Indiana University had been involved in making plans to institutionalize a new four-year program for elementary education majors in 1989. Two elements of this new program were directly related to the work of the *Preparing Elementary Teachers to Teach Mathematics Project* (*PETTM*). These two elements are:

1. each elementary education major must complete an 18-24 semester credit hour concentration in an approved area (one of those areas being mathematics); and
2. each prospective elementary education major must take part in a three semester sequence of concentrated early field experiences working in a school prior to student teaching.

The goals of the *PETTM* Project included:

1) better preparing all elementary school teachers through a problem-solving focus in newly developed mathematics courses at the university;
2) developing a program of concentration in mathematics consisting of some existing courses and some newly developed courses and experiences to prepare students who wish to make mathematics their field of concentration;
3) developing a close working relationship with the local school district to jointly plan for intensive early field experiences for all prospective teachers;
4) establishing support mechanisms to help practicing teachers in the school district broaden their views of the nature of mathematics and to help them utilize innovative teaching techniques for improving their pupils' mathematical thinking skills and use of problem-

solving strategies. As a result, local teachers should eventually be able to serve as role models in the teaching of mathematics using a problem-solving approach for the prospective teachers who visit their classrooms.

Background and Rationale

A number of national reports published in the mid-1980s called for the improvement of education at all levels. Three reports which attracted national attention were the NCATE Redesign (Gollnick & Kunkel, 1986), the Holmes Group report (1986), and the report of the Carnegie Task Force (1986). Based on the assumption that the quality of education is directly related to quality of teachers, these reports proposed dramatic changes in the practices used in recruiting, preparing, and certifying elementary school teachers. Both the Holmes Group and the Carnegie Task Force reports called for the elimination of undergraduate programs. All of the reports proposed broader and deeper backgrounds in liberal arts. The reports also called for closer ties and working relationships between teacher preparation institutions and school districts.

An elementary education Task Force at Indiana University studied the reports and, while seeing merit in many of their recommendations, declined to join the Holmes group. Although educators at Indiana University certainly agrees with the need for graduate work in education, they also feel that such study is more profitable after some teaching experience has been acquired. Further, the Task Force believed that it is quite possible to train elementary school teachers in a well designed four-year program (cf., Mehlinger, 1986). Thus, in 1989, Indiana University instituted their new four-year elementary education program, a program which incorporated many of the recommendations common to the reports mentioned above. Faculty in Mathematics Education and in Mathematics at Indiana University saw the revision of the teacher education program as a unique opportunity to improve the mathematics preparation of elementary teachers.

Project Description

The *Preparing Elementary Teachers to Teach Mathematics Project* was comprised of four components, all of which were designed to contribute to improving mathematics instruction in elementary schools: the *Content Component*, the *Methods Component*, the *School Component*,

and the *Research and Evaluation Component.*

The *Content Component* involved revision of the university mathematics courses required of PSTs in a fashion that could be exported to other teacher training institutions. Under the new Elementary Education program, prospective elementary teachers would take two mathematics courses: a finite mathematics course (M118—already required of freshmen intending many majors) , and a newly designed mathematics course specifically for education majors (T104). Under project auspices, the former course was restructured and the latter designed completely from scratch, so that problem solving is now a central focus in both. In other words, efforts are now being made to teach university-level mathematics with the same problem-solving approach that the Project would like to see teachers use when they work with children.

The *Methods Component* of the Project had three major goals: 1) to develop well-structured field experiences for both the existing mathematics methods course and a proposed new problem-solving methods course, 2) to develop and institutionalize the new problem-solving methods course (to be required, along with additional mathematics courses, of those students who would elect mathematics as their area of concentration), and 3) to develop a computerized problem-solving data bank to be available for use not only by students enrolled in methods classes and field experiences, but also by prospective teachers in other courses and local teachers in the schools.

An important part of the training of PSTs is experience with classroom teachers and their students. The purpose of the *School Component* of the Project was for Indiana University faculty to work together with faculty from a local elementary school (Dyer/Highland Park) to provide the best possible environment for early field experiences in mathematics for IU students. A full-time resource teacher worked closely with the Dyer/Highland Park School faculty for three years, helping them develop the confidence and expertise necessary to teach mathematics using a problem-solving approach. Five teachers from the school worked for two summers writing books of problem-solving materials that they, and their colleagues, could use with their students. The university-school partnership was a very successful part of the project.

The main purpose of the *Research and Evaluation Component* was to determine the

effectiveness of each of the other components of the project. A long range goal of the entire project was to provide an improved model for training elementary teachers. By finding out which innovations were effective, the project was able to provide a tested plan for training elementary teachers to teach mathematics.

Conclusion

The *Preparing Elementary Teachers to Teach Mathematics Project* was multifaceted, aiming to improve the teaching of mathematics in the elementary schools by encouraging teachers to use a problem-solving approach. Although the major focus of the Project was the design and implementation of a new program of studies for prospective elementary teachers at Indiana University, an essential element in this endeavor was the establishment of Dyer/Highland Park Elementary School as a site for all Project-related field experiences, and the provision of frequent and ongoing inservice work for the Dyer/Highland Park faculty so that school could serve as a model site for the teaching of mathematics with a problem-solving approach.

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PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS A PROBLEM-SOLVING APPROACH

OVERVIEW OF THE CONTENT COMPONENT

The *Content Component* involved revision of the Indiana University mathematics courses required of preservice teachers in a fashion that could be exported to other teacher training institutions. Under the new Indiana University Elementary Education program (effective in the Fall Semester, 1990), prospective elementary teachers would take two mathematics courses: an existing (three credit) finite mathematics course required of freshmen intending many majors (M118) , and a newly designed (four credit) mathematics course specifically for education majors (T104). Under project auspices, the former course was restructured and the latter designed completely from scratch, so that problem solving is now a central focus in both. In other words, efforts are being made to teach university-level mathematics with the same problem-solving approach that the Project would like to see teachers use when they work with children.

THE CONTENT COMPONENT: M118 (Finite Mathematics)

Finite mathematics courses are offered by most colleges and universities. These courses include topics such as set theory, counting methods, and basic probability theory, as well as linear systems, including linear programming. At Indiana University, the finite mathematics course provides a common mathematical background for over 4000 students per year. In particular, the course is taken by all students majoring in business and by most students majoring in the liberal arts and social and life sciences. Just prior to the beginning of this project, the Indiana University finite mathematics course, M118, was identified as a requirement for elementary education majors since it approaches several classical mathematics topics from a problem-solving point of view. Furthermore, the course consists of mathematics content believed to be fundamental for any college educated person, and it gives education majors a link with almost all students at Indiana University. The finite mathematics component of this project focused on revising the course so that it would not only serve the elementary education majors well, but would also better serve all college students taking the course.

Since the finite mathematics course plays such an important part in the training of undergraduates, there is great interest in the university community in the success of the course. For prospective elementary teachers it is especially important that the finite mathematics course provide a useful and stimulating experience, because this is the only mathematics course that most of them will take along with students from other majors. Accordingly, some of our work was a joint project with the mathematics department to ensure that the finite mathematics course would

meet the needs of all students required to take it -- most particularly, elementary education majors. The project hired a graduate assistant and an undergraduate computer programmer to work on various tasks related to M118. The finite mathematics component of the project was active on several projects. Following is a summary of these projects and the progress that was made:

1. Syllabus Modification. The syllabus for M118 was changed to decrease the time devoted to business related topics and to increase the time devoted to topics traditionally associated with the social and life sciences. In particular, the simplex method is no longer included in the syllabus and the topic of Markov chains is now a significant part of the course. The new syllabus was first used during the academic year 1988-89, and based on that experience, it was modified slightly for 1989-90. It was used again during 1990-91 and 1991-92, and is expected to remain essentially unchanged in the foreseeable.

2. Computer-based Assistance. Computer-assisted lessons were developed to assist students who need additional practice, help, and instruction outside of class. Four computer-based lessons, which had been created during a mathematics department pilot project, were extensively tested and de-bugged. These lessons were tested with several sections of the course and were used very successsfully by many students. An additional lesson (on Markov chains) was written and programmed during the summer of 1989. To get the elementary education majors who were taking M118 involved with the computer lessons, a special help session was held particularly for them one night each week during the spring semester of 1990. Even though more than one personal letter was sent to each of these students announcing the special help session, the turnout was small. However, those who came to these special sessions provided very positive remarks about the computer lessons.

There were some disappointments throughout the duration of the project, however, concerning the development and implementation of the computer lessons. The original four lessons were written for the Indiana University mainframe computers and required a terminal with special graphics capabilities. The Indiana University computing center could not provide enough of these terminals to serve the nearly 2000 students enrolled in the course each semester. Several efforts were made to remedy this problem, including translating the lessons to a new language, so that they could be used on PCs as well as on the Indiana University mainframe. Unfortunately, the computer center did not present a conversion plan, and in particular, an acceptable language for the translation until the project was nearly completed. The lessons are still available on the mainframe computers, however, and they are being used by some students each semester.

3. Test Generator. To help coordinate the many sections of the Finite Mathematics course, a test generator, consisting of over 1200 questions, was created for the use of instructors. It was used on a trial basis during the 1989-90 year and made available for all instructors starting with the 1990-91 academic year.

4. Tool Kit. Work was done on developing on a computer toolkit to help students with

homework problems and to aid them in checking their own work. One unit of the tool kit was completed, namely the unit which helps with the process of solving systems of linear equations. Other units involve matrix algebra and Markov chains.

5. Videotaped Lessons. A student guide, including practice problems, was prepared for four short videotapes, which had been created prior to the project. These tapes allow students to review a key idea covered in class at their own pace and convenience. Several students successfully used the videotapes and accompanying guide during the summer of 1990, and they have been available to students since then.

Additionally, the graduate assistant hired to work on this component of the project did his dissertation research concerning the errors that students make throughout a finite mathematics course. One of the results of this research was a list of the frequency of errors made on the 23 quizzes given in the fall 1990 section of M118 taught by Professor Maki. This list could be helpful to M118 instructors in the future.

During spring, 1989, elementary education majors were encouraged for the first time to begin enrolling in M118 (on a voluntary basis, substituting it for one of their presently required mathematics course for prospective elementary teachers). Fourteen education students enrolled in M118 in a section taught by Daniel Maki, and the range of grades achieved by them was quite similar to those achieved by non-education majors. Effective with the fall semester of 1989, all new students who planned to be elementary education majors were advised to take M118 as their first mathematics course. As a result more sections of M118 were offered during 1989-90, and the mathematics courses previously offered specifically for education majors were in the process of being phased out. During fall, 1989, about 90 prospective elementary education majors enrolled in M118.

The process of integrating all elementary education majors into the new program has essentially been completed. During the fall semester of 1990, all new prospective elementary education majors were advised to take Finite Mathematics during their first semester on campus. Of the 151 prospective elementary education majors who were enrolled in the course in fall 1990, 16% received grades of A or B, 41% received a C, 22% received grades of D or F, and 21% withdrew before midterm. While this achievement is somewhat below average for all students enrolled in the course, the prospective teachers also scored somewhat lower on the 26-problem university-entry matheamtics skills assessment test. The non-education majors enrolled in M118 scored an average of 14.6 on this test, and the prospective elementary education majors enrolled in M118 scored an average of 13.5. The prospective teachers averaged 925.0 on the Scholastic Aptitude Test, and they averaged a high school class rank of 21st out of 100 students. A sizable amount of data were collected on this group of prospective elementary education majors. These students have continued their mathematical training by taking our new course T104.

PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS: A PROBLEM-SOLVING APPROACH

THE CONTENT COMPONENT: T104
(Mathematics for Elementary Teachers via Problem Solving)

Goals of T104

From the beginning, the primary goal for this aspect of the content component of this project has been to develop a new undergraduate course for preservice elementary teachers (PSTs) with problem solving and exploration at its core. A subsidiary goal has been to institutionalize this course at Indiana University - Bloomington (IU - B). Both of these goals have been attained: the course exists and is now one of two required mathematics courses for PSTs at IU - B.

In the addendum to the original proposal for this project, we noted: "To state that problem solving will be integrated throughout the course is an ambitious goal. We do not intend simply to identify several clever and interesting problems to sprinkle liberally among topics. . . . Although some existing textbooks on mathematics for elementary teachers have made a serious attempt to make problem solving a focus in the treatment of topics, all of them fall far short of the mark" (Proposal Addendum No. 87-51478, p. 6). Thus, we suspected from the outset that the development of the sort of course we were envisioning would not be an easy task to accomplish. Our suspicious were well founded: the task proved to be even more difficult than we had imagined. Three factors served to make our efforts especially challenging:

1. It is very difficult to expose PSTs to all the mathematical ideas we view as important in a one-semester course which emphasizes problem solving in small cooperative groups;
2. Many (in some sections, most) PSTs were actively opposed to the spirit of the course in the beginning (they simply were not used to being asked to think about mathematical ideas);
3. The course was taught by graduate students who often were neither sensitive to the professional needs of elementary teachers, nor intellectually and emotionally prepared to "teach" mathematics via problem solving.

Notwithstanding these obstacles, the course was developed and we think it is deserving of its official title: *Mathematics for Elementary Teachers Via Problem Solving*. Better known as T104, this new course has had four main goals since its inception:

1. To help students develop adult-level perspective and insights into the nature of mathematics taught in the elementary school;
2. To improve students' ability to engage in mathematical thinking and reasoning;
3. To increase students' ability to use their mathematical knowledge to solve problems; and
4. To expose students to learning mathematics via problem solving.

History of the Development of T104

During the Fall Semester of 1988 a task force was formed to plan for and design mathematics laboratory activities for the new course, T104. From January until May 1989 the task force tried out many of the proposed activities with groups of undergraduate volunteers. Reception of the laboratory-based lessons by these students was favorable, but it was clear that substantial revision was needed of most of the activities and that some of the activities probably were not suitable for our purposes. Summer 1989 work involved modifying and refining the activities already designed, designing additional activities, and collecting and modifying information to be included in the student and teacher materials.

In the Fall Semester of 1989, the 250 articles and activities collected during the summer were sorted and sets of activities were developed on the following content areas: numeration, operations, geometry, measurement, number theory, and rational numbers. Teachers notes were also developed to accompany the activities. Some of the number theory and rational numbers activities were tried out in an existing course for prospective elementary teachers. Videotapes were developed of PSTs working in small groups on problems of the sort that are included in the course.

During the Spring Semester 1990 (January-early May), there were two pilot sections of the course, each section having about 25 students. During this semester, a trial of a complete set of the instructional materials was initiated. As they used the materials throughout the semester, the two instructors kept extensive notes with respect to such factors as: level of difficulty for students; adequacy of the directions, explanations, etc.; appropriateness for use in a lab setting; and student satisfaction. In addition, videotapes were made of "experts" modeling problem solving in front of the class. Assessment procedures that were aligned with the curriculum were developed during this semester, including problems used in group testing situations. Reflective writing assignments and out-of-class projects were other tasks worked on during this semester.

Substantial revisions were made during the summer months and an "abbreviated" set of teachers' notes was prepared in anticipation of offering five sections (about 150 students total) of T104 during the Fall Semester 1990 (late August-December). Four orientation sessions were held during Summer 1990 to acquaint the Fall instructors with the philosophy and overall nature of the course. Not only were the revised instructional materials put to the test during the Fall Semester, but also the abbreviated teachers' notes were used for the first time. All of the five sections were taught by teachers who had not taught the course previously, nor had they been involved in the development of any of the activities. However, an experienced T104 teacher, who had also helped develop many of the activities, was employed as the course coordinator. She was able to further orient the new instructors, provide feedback about the course, and aid in the development of tests. More out-of-class projects were developed during this semester.

By employing instructors who were completely new to the course, we hoped to be able to gauge the adequacy of the teachers' notes. Furthermore, each section of the course was visited by an experienced observer once a week for purposes of assessing the appropriateness and effectiveness of the revised instructional materials. Weekly debriefing sessions were held with the instructors in order to get their reactions to the notes and materials. The observations and debriefing sessions provided numerous ideas for further revisions, particularly of the teachers' notes. Videotaping of instructors and small groups of students working on T104 activities took place in an effort to develop some tapes to be used for new instructor orientation.

In Spring Semester 1991 there were five new sections of T104 (about 150 students with five instructors, two of whom were new). Additional, but minor, revisions of the instructional materials and teachers' notes were conducted and a bank of test problems was developed for use by the instructors. A tests section and a projects section were developed for the teacher notes. These explained the basic format of the tests and projects and provided guidance for developing tests and new out-of-class projects. One additional out-of-class project was developed bringing the total to 14. Videotaping of instructors and small groups of students continued throughout the semester. Post-unit meetings were again help with the instructors to get their reactions to the teacher notes and student activities.

In the Summer of 1991 final revisions of all materials took place. In addition, introductory sections were developed for both the teacher materials and the student materials. For the teacher materials, this section contains information about such topics as the goals and philosophy of T104, problem solving, the teacher's and students' roles in T104, cooperative learning, assessment, engaging the students in reflectiveness, the teacher notes, and the need for and use of a textbook.

The introductory section in the student materials has the following information about T104: purpose and organization of a student notebook, information about assessment and grading, information about cooperative work, Course Schedule, Materials Schedule, Homework Schedule, Problem-Solving Tips sheet, and Notebook Grading Sheets. Finally, a narrated videotape appropriate for orienting new instructors to the course was developed. This tape shows excerpts from actual classes of teachers "introducing an activity," "commenting to small groups while they work," "leading class discussions," and "wrapping up after small group work." The video tapes of an expert modelling problem-solving behavior and of small groups solving a problem from the T104 materials are used in the T104 classroom to allow the students to observe other persons solving problems. A discussion follows the viewing of each videotape, focusing on the ways the person(s) on the tape tried to understand the problem, make and implement a plan, and evaluate the solution.

Philosophy of T104

The nature of mathematics instruction has changed rather dramatically in recent years. In the past, mathematics instruction was viewed by many as an activity in which an "expert" attempted to transmit her or his knowledge of mathematics to a group of mostly passive students. This passive transmission view has been replaced by one involving students as active participants in the act of "doing"—that is, creating, exploring, testing, verifying, etc.—mathematics. This new vision is one in which mathematics is seen as a cooperative venture in which students are encouraged to explore, make and debate conjectures, build connections among concepts, solve problems growing out of their explorations, and construct personal meaning from all of these experiences.

T104 was designed to reflect this pedagogical shift. The emphasis in the course is on learning mathematical concepts and processes through solving problems. Moreover, it is our conviction that problems are best solved in an environment of cooperation among small groups of students in which the teachers plays the roles of guide, coach, question asker, and co-problem solver.

Six beliefs about mathematics instruction have been central to our efforts to develop a new course for PSTs, and it is safe to say that they are at the heart of the philosophy of T104 stated above. To a large extent these beliefs have evolved over a rather long period of time (at least since 1972 -1977 during the days of an earlier NSF-sponsored project, The Mathematics Methods Project). These six beliefs are as follows:

1. In order for students to improve their ability to solve mathematics problems, they must attempt to solve a variety of types of problems on a regular basis and over a prolonged period of time.
2. There is a dynamic interaction between mathematical concepts and the processes (including metacognitive ones) used to solve problems involving those concepts. That is, heuristics, skills, control processes, and awareness of cognitive processes develop concurrently with the development of an understanding of mathematical concepts.
3. Problem-solving instruction, metacognition instruction in particular, is likely to be most effective when it is provided in a systematically organized manner under the direction of the teacher.
4. Problem-solving instruction that emphasizes the development of metacognitive skills should involve the teacher in three different, but related, roles: (a) as an external monitor, (b) as a facilitator of students' metacognitive awareness, and (c) as a model of a metacognitively-adept problem solver.
5. The standard arrangement for classroom instructional activities is for students to work in small groups (usually groups of four). Small group work is especially appropriate for activities involving new content (e.g., new mathematics topics, new problem-solving strategies) or when the focus of the activity is on the process of solving problems (e.g.,

planning, decision making, assessing progress) or exploring mathematical ideas..

6. The teacher's instructional plan should include attention to how students' performance is to be evaluated. In order for students to become convinced of the importance of the sort of behaviors that a good problem-solving program promotes, it is necessary to use evaluation techniques that reward such behaviors.

What is Problem Solving in Mathematics and How Should It Be Taught?

All mathematics educators agree that problem solving is a very important, if not the most important, goal of mathematics instruction at every level. Indeed, some have even gone so far as to insist that: "Problem solving should be the focus of school mathematics" (National Council of Teachers of Mathematics, 1980, p. 1), and "Problem solving is a central focus of the mathematics curriculum" (Mathematical Sciences Education Board, 1990, p. 31). Since about 1980, problem solving has been the most written about, but possibly the least understood, topic in the mathematics curriculum in the United States. It is probably safe to say most teachers agree that the development of students' problem-solving abilities is a primary objective of instruction. Although acceptance of the notion that problem solving should play a prominent role in the curriculum has been widespread, there has been anything but widespread acceptance of how to make it an integral part of the curriculum. To date no mathematics program has been developed that adequately addresses the issue of making problem solving the central focus of the curriculum. Instead of programs with coherence and direction, teachers have been given a well-intentioned *melange* of story problems, lists of strategies to be taught, and suggestions for classroom activities. If problem solving is to become a more prominent goal of mathematics instruction, more serious and thoughtful attention must be given to what it means to make problem solving the focus of school mathematics.

Why Is Mathematical Problem Solving Difficult for Students?

It is our view that many students are unable to solve any but the most routine problems for three basic reasons: (1) problem-solving is an extremely complex form of intellectual activity; (2) the process of problem solving has been viewed much too simplistically; and (3) students are given too few opportunities to engage in real problem solving. In the following sections we discuss each of these reasons.

The Complex Nature of Mathematical Problem Solving[1]

Problem solving is an activity requiring the individual to engage in a variety of cognitive actions, each of which requires some knowledge and skill, and some of which are not routine. Furthermore, these cognitive actions are influenced by a number of non-cognitive factors. That is

[1]A more thorough discussion of these factors is found in a paper by Lester, Garofalo and Kroll (1989).

to say, by its very nature problem solving is an extremely complex form of human endeavor that involves much more than the simple recall of facts for the application of well-learned procedures. Although it is difficult to define problem solving, the following statement comes close to capturing our sense of what it involves: *Successful problem solving involves the process of coordinating previous experiences, knowledge, and intuition in an effort to determine an outcome of a situation for which a procedure for determining the outcome is not known.* Put another way, problem solving is taking place when the individual (or group) is not sure that what they are doing will result in the solution. Moreover, the ability to solve mathematics problems develops slowly over a very long period of time because it success depends on much more than mathematical content knowledge. Problem-solving performance seems to be a function of at least five broad, interdependent categories of factors: (1) knowledge acquisition and utilization, (2) control, (3) beliefs, (4) affects, and (5) socio-cultural contexts. These five categories overlap (e.g., it is not possible to completely separate affects, beliefs, and socio-cultural contexts) and they interact in a variety of ways too numerous to name in these few pages (e.g., beliefs influence affects, and they both influence knowledge utilization and control; socio-cultural contexts have an impact on all the categories). It is perhaps due to the interdependence of these categories that problem solving is so difficult for students.

Two Models of the Process of Solving Problems[2]

It is our view that students' inability to solve problems stems in large part from the fact that problem solving has often been conceptualized in a simplistic way by a model like the one shown in Figure 1. This model has two levels or "worlds": the everyday world of things, problems, and applications of mathematics and the idealized, abstract world of mathematical symbols and operations. In this model the problem-solving process has three steps. Beginning with a problem posed in terms of physical reality, the problem solver first translates (arrow A) the problem into abstract mathematical terms, then operates (arrow B) on the mathematical representation in order to come to a mathematical solution of the problem, which is then translated (arrow C) into the terms of the original problem. According to this view, mathematics may be and often is learned separately from its applications. Teachers who adhere to this model are very concerned about developing students' skills with the processes of translating real world problems into mathematical representations and vice versa. But they tend to deal with problems and applications of mathematics only after the mathematical concepts and skills have been introduced, developed, and practiced. The difficulty with this model is that it applies to routine problems better than to non-

[2]This and the section that follow it are slightly modified versions of portions of a longer paper written by Schroeder and Lester (1989).

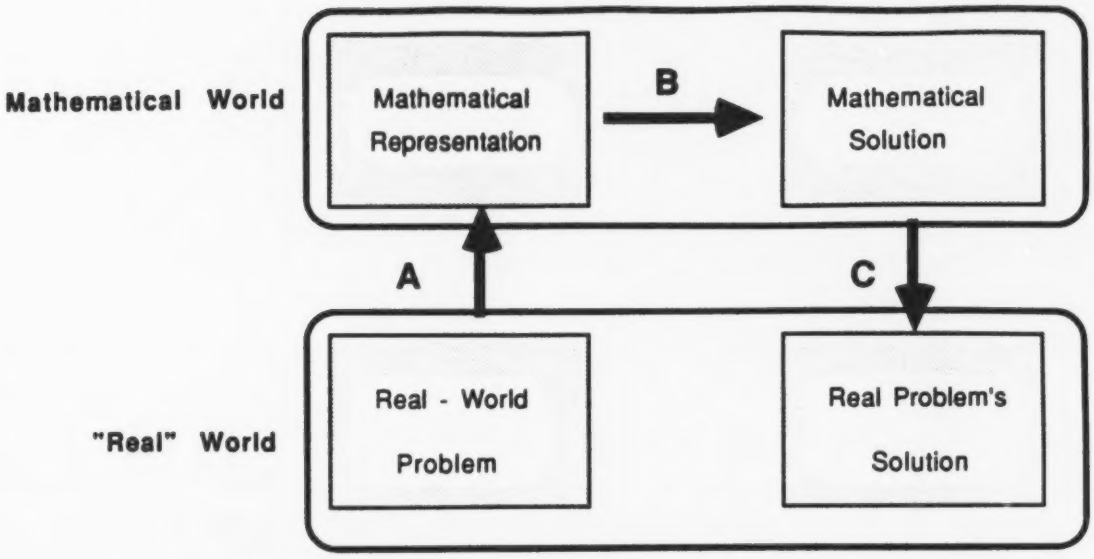

Figure 1. A Simplistic Model of the Process of Solving Mathematics Problems

routine ones. Problems like those that have been classified as "translation problems" (Charles & Lester, 1982) are solved exactly as the model indicates. But for more challenging problems (like those Charles & Lester categorize as "process problems") the problem solver has no already-learned mathematical operation that will solve the problem. In addition to translation and interpretation, these non-routine problems also demand more complex processes such as planning, selecting a strategy, identifying sub-goals, conjecturing, and verifying that a solution has been found. For non-routine problems, a different type of model is required.

Figure 2 shows a modification of the problem-solving model for translation problems that can be used to illustrate thinking processes when non-routine problems are involved. Like the previous one, this model also contains two levels representing the everyday world of problems and the abstract world of mathematical symbols and operations. In this model, however, the mathematical processes in the upper level are "under construction" (i. e., being learned, as opposed to already learned) and the most important features are the relationships between steps in the mathematical process (in the upper level) and actions on particular elements in the problem (in the lower level).

In the figure, some of the Yi arrows point upward to indicate that the problem solver is learning to make abstract written records of the actions which are understood in a concrete setting. These upward-pointing arrows represent the processes of abstraction and generalization. And,

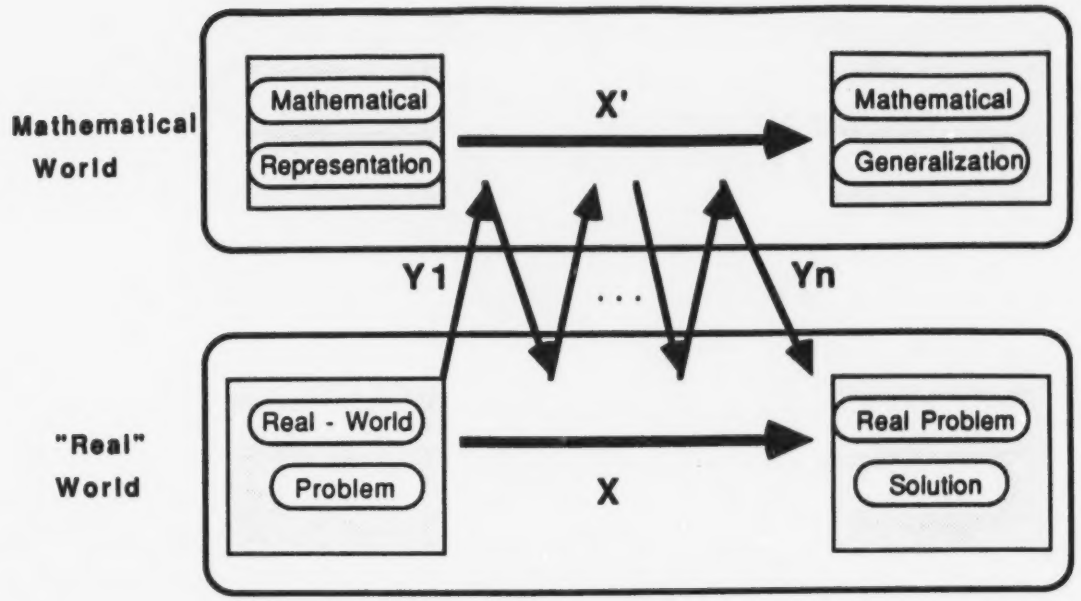

Figure 2. A Modified Model of the Process of Solving Mathematics Problems

some of the arrows point downward to show that the problem solver is able to explain a mathematical process by referring to the real world actions that the mathematics represents. A downward pointing arrow might also be used to suggest that a problem solver who had forgotten the details of a mathematical procedure would be able to reconstruct that process by imagining the corresponding concrete steps in the world in which the problem was posed. The collection of Yi arrows taken together illustrate that typically the problem solver moves back and forth between the two worlds -- the real and the mathematical-- as the need arises. It may happen that for a particular problem the problem solver is able to move directly from the real world along arrow Y1 to the mathematical world and proceed along arrow X' directly to a mathematical generalization and hence to a solution to the original real world problem (In such a case the solution process can be modelled as shown in Figure 1).

Giving Students Opportunities to Engage in Real Problem Solving

For students who are struggling to become better problem solvers the difficulty caused by the complexity of problem solving is compounded by the fact that most of them do not receive adequate instruction, either in quality or quantity. Unfortunately, we do not yet have fool-proof, easily followed and implemented methods of helping students to improve their problem-solving ability. Thus, the third reason of confusion about the role of problem solving in the school mathematics curriculum concerns the lack of agreement as to how to make problem solving the focus of instruction. One of the best ways we have found for coming to grips with these

differences is to distinguish among three approaches to problem-solving instruction: (1) teaching *about* problem solving, (2) teaching *for* problem solving, and (3) teaching *via* problem solving. (An explicit statement of this distinction was contained in a paper written more than a decade ago by Larry Hatfield (1978), but we suspect that others may have espoused a very similar point of view as well.) Let us discuss what each of these three approaches entails.

Teaching ABOUT Problem Solving. The teacher who teaches *about* problem solving highlights George Polya's model of problem solving (or some minor variation of it). Briefly, Polya's model describes a set of four interdependent phases that are involved in the process of solving mathematics problems: understanding the problem, devising a plan, carrying out the plan, and looking back. Students are explicitly taught the phases, which according to Polya, expert problem solvers use when solving mathematics problems, and they are encouraged to become aware of their own progression through these phases when they themselves solve problems. Additionally, they are taught a number of "heuristics" or "strategies" from which they can choose and which they should use in devising and carrying out their problem-solving plans. Some of the several strategies typically taught include: looking for patterns, solving a simpler problem, and working backwards. At its best, teaching *about* problem solving also includes experiences actually solving problems, but it always involves a great deal of explicit discussion of and explicit teaching about how problems are solved.

Teaching FOR Problem Solving. In teaching *for* problem solving the focus is on ways in which the mathematics being taught can be applied in the solution of both routine and non-routine problems. Although acquiring mathematical knowledge is of primary importance, the teacher interested in teaching *for* problem solving thinks the essential purpose for learning mathematics is to be able to use it. Consequently, students are given many instances of the mathematical concepts and structures and many opportunities to apply mathematics in solving problems. Further, the teacher who teaches *for* problem solving is very concerned about students' ability to transfer what they have learned from one problem context to others. A strong adherent to this approach would suggest that the sole reason for learning mathematics is to be able to use the knowledge gained to solve problems.

Teaching VIA Problem Solving. In teaching *via* problem solving, problems are valued not only as a purpose for learning mathematics, but also as a primary means for doing so. The teaching of a mathematical topic begins with a problem situation which embodies key aspects of the topic, and mathematical techniques are developed as reasonable responses to reasonable problems. A goal of learning mathematics is to move certain types of problems from non-routine status to routine status. The learning of mathematics in this way may be viewed as a development from the concrete (a "real world" problem which serves as an instance of the mathematical concept or

technique) to the abstract (a symbolic representation of a class of problems, and techniques for operating with those symbols).

Some Observations about the Three Approaches. Although these three conceptions of teaching problem solving in mathematics can be isolated in theory, in practice they overlap and occur in various combinations and sequences. Thus, it is probably counter productive to argue in favor of one or more of these types of teaching or against others. Nevertheless, if an instructor intends to make problem solving the "focus of instruction," he or she needs to be aware of the limitations inherent in exclusive adherence to either of the first two types of problem-solving instruction. One such limitation stems from the fact that problem solving is not a mathematics topic and it should not be regarded as such. If teaching *about* problem solving is the focus, there is a danger that "problem solving" will be regarded as a strand to be added to the curriculum. Instead of problem solving serving as a context within which mathematics is learned and applied, it may become just another topic taught in isolation from the content and relationships of mathematics.

A different kind of shortcoming can arise from teaching *for* problem solving. When this approach is interpreted narrowly, problem solving is viewed as an activity students engage in only after the introduction of a new concept or following work on a computational skill or algorithm. The purpose of this approach is to give students an opportunity to "apply" the concepts and skills recently learned to the solution of "real world" problems. Often, these problems appear under a heading such as "Using Division to Solve Problems" and a solution of a sample story problem is provided as a model for solving other, very similar, problems. More often than not solutions to these problems can be obtained by simply following the pattern established in the sample and when students encounter problems that do not follow the sample they often feel at a loss. It has been our experience (which is supported by several studies) that students taught in this way often simply pick out the numbers in each story and apply the given operation (s) to those numbers without regard for the problem's context and, as often as not, they obtain the correct answers. Simply put, we do not regard this sort of activity as problem solving. Indeed, it does not even involve mathematical thinking. Furthermore, a side effect is that students learn that all mathematics problems can be solved quickly and relatively effortlessly, and without any need to understand how the mathematics they are using relates to real situations. Unfortunately, this approach to problem-solving instruction has been quite common in textbooks at all levels -- elementary through college.

Unlike the other two approaches, teaching *via* problem solving is a conception that has not been adopted either implicitly or explicitly by many teachers, but it is an approach to the teaching of mathematics that deserves to be considered, tried out, and evaluated. Indeed, teaching *via* problem solving is the approach that is most consistent with the recommendation of the NCTM's Standards Commission that: (1) mathematics concepts and skills be learned in the context of solving

problems; (2) the development of higher level thinking processes be fostered through problem-solving experiences; and (3) mathematics instruction take place in an inquiry-oriented, problem-solving atmosphere (NCTM, 1989).

The point of the foregoing discussion is that progress has been slowed tremendously because there has been no commonly accepted view of what it means to teach with a problem-solving perspective. This lack of consensus has hampered researchers as well as curriculum developers and teachers.

Important Components of T104

Cooperative Learning

The philosophy of T104 requires that students in the course—rather than the teacher— must bear the primary burden for constructing mathematical arguments and for providing mathematical explanations. Shifting the burden of explanation in this way is accomplished by organizing the T104 class in a very different way from the usual college mathematics class: during much of each class period T104 students work cooperatively in small groups to wrestle with problems that challenge them, to develop new and deeper understandings of fundamental mathematical concepts, and to talk about their new ways of thinking.

Forming Cooperative Groups

On the first day of class the instructor will need to group students into cooperative groups of about four students each. Since most students will not yet know each other, initial groups can be formed by merely suggesting that four students who are sitting near one another put their desks together for small-group work. Students will then be expected to keep working each day with the same group of people until the teacher indicates that it is time for a change of groups (usually every 3 or 4 weeks).

When the time comes, some instructors find it best to form new groups totally at random. One way to do this is to put a pile of playing cards near the door and to instruct students to take a card as they arrive at class on the day when groups are to be changed. Then those four who have the same card number (for example, 7 or 2 or Queen), regardless of suit, form a group. (The instructor will have to take extra cards out of the deck ahead of time so that the appropriate number of groups of four cards remain—for example if there are 24 students in the class, an instructor might remove from the deck all cards except those from Ace through 6. This will leave 24 cards, six sets of four cards each.)

There may be times when random assignment to groups does not seem appropriate. For example, if there are one or two students in a class who have trouble working together, the instructor may want to assign groups so that these students are separated. Or the instructor may want to arrange groups so that neither the best students (nor the weakest students) are concentrated in one group. Near the end of the semester, an instructor may choose to ask each student to turn in

a list of individuals they would like to work with and may form groups deliberately to include students who have expressed an interest in working together.

Instructor Behaviors During Small Group Work

The role of the instructor in T104 is quite different from in more traditional mathematics classes. Rather than preparing a lecture, the T104 instructor is usually responsible for a three-part lesson: 1) providing a brief introduction to the day's activities, 2) circulating about the room while students work in small groups and making appropriate comments to the groups, 3) leading wrap-up whole-class discussions where various groups share their thinking about the problem and the instructor helps everyone to consolidate their thinking about their work. [This type of wrap-up often occurs several times throughout the class period, as well as at the end of each class.]

Providing a Brief Introduction. Before students begin work in their groups, the instructor should talk briefly to the entire class, introducing the activity of the day, explaining any new terminology or special instructions, and indicating how this activity fits in to the larger context of the course. Note that *this introduction is not a time for telling how to solve the problem* at hand: solving the problem is the task of the students in their small groups. T104 students will probably be frustrated at first with small group work because they are accustomed to being told by their teachers exactly what to do. By contrast, T104 problem-solving activities challenge students to do their own thinking and the instructor's role is merely to introduce the activity and to guide students to discovering their own solutions.

Circulating While Students Work in Small Groups. As you circulate around the room while students are working in small groups, they need to understand that you will play the role of question asker, problem poser, and careful listener, but that you expect them to be the problem solvers and the explainers. *You can tell your students over and over that they must be the problem solvers; but they will only believe it if you demonstrate it by your actions in the classroom.*

- *Don't be an answer giver.* Try not to provide right/wrong judgments or to tell students how to proceed on a problem. If you do, they will always be waiting for you to tell them the next time, instead of thinking for themselves. If students ask, "is this right?" ask them how they might decide for themselves. Some appropriate replies are "what do your group members think?" or "can you find another way to verify your answer?"
- *Deemphasize correct answers.* Try to help students understand that you are more interested in depth of understanding, in ability to communicate mathematical ideas clearly, and in reasonable thinking than simply in correct answers. As you move from group to group, avoid asking "what did you get?" Instead ask questions that require explanation such as "can you tell me what you've been thinking?" or "what strategies have you been using to approach this problem?"

- *Be prepared with appropriate hints.* There is more than one way to solve nearly every problem. When students are stuck, the instructor may need to provide a hint, but the hint should build on whatever progress the group has already made. For example, if students have been experimenting with specific numbers, it might be appropriate to suggest organizing findings in a table so that a pattern may become more apparent. If students are stuck because a problem has very large numbers or seems too complicated, it might be good to suggest trying some simpler cases first. It is usually not appropriate to provide a hint that merely provides students with the first (or a subsequent) step in a problem solution because this is often tantamount to telling students how to solve the problem.
- *Be prepared with problem extensions.* Some groups in the classroom will work faster than others. The experienced instructor helps these students (and eventually all students) to think beyond the task at hand. For example, it is often useful to ask students how they would solve the problem if conditions were changed (numbers different, question different, more constraints, etc.) or to ask them if they can work from their specific solution to a solution for a generalization of the problem.

Wrapping Up after Small-group Work. One of the most important parts of the T104 class is the wrap-up discussion that takes place after small groups have worked on problems. Once again, the instructor must guard against playing too prominent a role. There is no point in telling the class how their groups should have solved the problem. An instructor who does this will find that groups soon have no motivation to work on their own: why should students struggle to work a problem if they know the teacher will explain it later? The wrap-up discussion should be a session where all groups have a chance to tell what approaches they tried, how successful or unsuccessful they were, and what conclusions they drew from their efforts. The best discussions are those in which the students do most of the talking—comparing approaches, arguing, and trying to convince one another of the validity of their findings. The instructor's role is to orchestrate this discussion so that everyone has an opportunity to participate, so that everyone can hear and understand what others are saying, and so that some closure is reached by the end of the discussion. Although it is important to allow everyone to contribute (even those whose solutions are incorrect), it is also important not to leave students hanging at the end, uncertain what makes sense and what doesn't. At the end of an ideal problem discussion, students will have reached their own conclusions about the validity of various problem solutions and will have a good sense of how the day's work fits into the bigger picture of the mathematical concepts being studied in the course.

Big Ideas in Mathematics for Preservice Elementary Teachers

The term " big idea" refers to those themes that pervade several areas of mathematics and which serve to make connections among mathematics topics. These big ideas, then, are recurring,

unifying themes in mathematics. With respect to identifying KEY big ideas to emphasize in a course or courses for prospective elementary teachers (PSTs), we determined that it does not matter a great deal which ones are chosen. Instead, what matters is that to be a big idea an idea should:

1. Help PSTs make connections among apparently (for them) unrelated topics;
2. Help PSTs make connections between the world of mathematics and the "real world," and
3. Provide ample opportunities for PSTs to "meet the Queen" (i.e., come to have a greater appreciation for and understanding of the beauty and subtlety of mathematics and mathematical activity).

Notwithstanding our belief that it is not so important to be concerned about selecting a specific set of themes, we thought it would be useful to take a look at what others think the big ideas are. In particular, we found the views of Davis and Hersh (in *The Mathematical Experience*), Steen and others (in *On the Shoulders of Giants*), and the authors of the 24th yearbook of the NCTM (in *The Growth of Mathematical Ideas: Grades K - 12*) to be especially helpful.

The Mathematical Experience (Davis & Hersh, 1981)

Davis and Hersh identify both *inner* and *outer* issues related to doing and studying mathematics. Their inner issues seem to be closest our notion of big ideas. They are as follows: Abstraction, Aesthetics, Algorithmic vs. Dialectic Mathematics, Formalization, Generalization, Infinity, Math Objects & Structures, Pattern, Order & Chaos, Proof, Symbolism, Unification

On the Shoulders of Giants (Steen, 1990)

The authors of this recently published treatise on the nature of contemporary mathematics discuss two types of ideas: *connective themes* and *deep ideas* (that nourish the branches of mathematics). Both types seem to be related to our view of big ideas. Their connective themes include: algorithms, classification, exploration, inference, measurement, symmetry, and visualization. Among their deep ideas range are the notions of: mathematical structures, attributes, actions, abstractions, attitudes, behaviors, and dichotomies.

The Growth of Mathematical Ideas: Grades K - 12 (NCTM, 1959)

The authors of the 24th yearbook of the NCTM insisted that "teachers . . . plan so that pupils continually have recurring but varied contacts with the fundamental ideas and processes of mathematics" (p. 2). Their list of big ideas includes the following: language and symbolism, mathematical modes of thought, measurement and approximation, number and operation, probability, proof, relations and functions, and statistics.

Our Current Thoughts about Big Ideas

Of course, each of the authors considered above used somewhat different terminology, but there was considerable agreement that the number of truly big ideas is relatively small. Moreover,

it is interesting to note that traditional branches of mathematics (e. g., algebra, geometry, calculus) were not viewed by any of the authors as being big ideas in themselves and mathematical "actions" seemed to be as important as mathematical "structures."

Our own thinking, guided by a consideration of the ideas expressed in the three volumes mentioned above, has led to a conceptualization of the big ideas in terms of an m x n x p matrix: *structures* x *actions* x *tactics* (m, n, and p are not necessarily distinct). Each dimension contains big ideas that exemplify a fundamental theme, process, or aim of mathematical activity.

Structure includes mathematical ideas and entities such as *equivalence, function and relation, measurement, number and operation, shape and space*, and *pattern and order*, among others. The structures identified here are almost arbitrary; many other (perhaps better) structures could have been chosen. This is consistent with the notion that the specific big ideas are not nearly as important for these courses as the idea that there are unifying themes that pervade many branches of mathematics.

The *actions* refer to the sorts of activities that individuals engage in when they are doing mathematics and, as often as not, they are the goals of mathematical activity (e.g., creating a generalization, developing a useful mathematical model). These actions help distinguish mathematical activity from other kinds of activity. Among the actions that seem to be particularly relevant for courses for PSTs are *verifying, generalizing, modelling, representing*, and *composing/decomposing*.

The *tactics* dimension includes those generic tools that aid the individual in doing mathematics. The four tactics we have chosen are *conjecturing*, creating and using *algorithms*, using *problem-solving heuristics*, and developing and using appropriate *language and symbolism*, but there are probably other equally as important tactics.

Finally, we hasten to add that this matrix is tentative, incomplete, and probably under-conceptualized at this time. However, we think it can provide a framework for future course

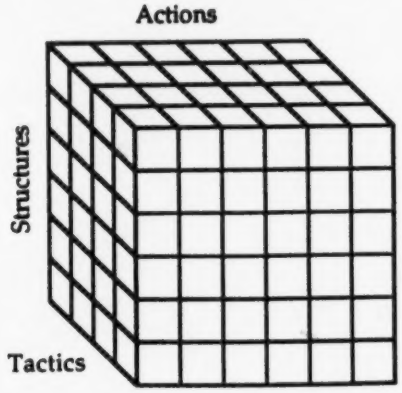

Figure 3. *Structures* x *Actions* x *Tactics* Matrix of Big Ideas

Reflective Writing

Reflective thinking occurs inside and outside the T104 classroom. In the classroom, students reflect as they work to understand a problem, evaluate their solution processes, decide if their solutions make sense, justify their generalizations, connect mathematical concepts, understand a problem solution different from their own, extend a problem, monitor their thinking processes, and communicate their ideas to other students and the teacher. When students are encouraged and expected to be reflective in their work, they become better at thinking reflectively, their understanding of the content improves, they are more creative and insightful in their problem solving, their motivation for learning increases, and they begin to look for, and make, connections between mathematical concepts.

Outside of the classroom students can continue their reflective activity through reflective writing. Reflective writing benefits students in four ways: (1) therapeutic value, (2) increased learning of content, (3) improvements in learning and problem-solving skills, and (4) change in one's conception of mathematics. Reflective thinking and writing are meaning-making processes that involve the learner in actively building connections between what she's learning and what is already known.

In general, there are two broad types of reflective writing assignments that are used in T104 to give students the opportunity to reflect on their feelings and thoughts and communicate these in written form. The first type is an assignment that encourages the students to come to terms with their feelings and beliefs about mathematics. Some examples of this type are as follows:

- *Write a brief description of your thinking as you played the games "Poison" and "What's My Number?"*
- *Discuss how you think mathematics fits with the real world. How do you think new mathematics is created? Has this changed your definition of mathematics?*

The second type of reflective writing assignment asks the students to explain a mathematical idea or procedure. It forces the students to ask themselves, "Do I really understand this?" Several examples of open-ended assignments of this type are listed below.

- *Explain why the Sieve of Eratosthenes works, in general, in identifying prime numbers.*
- *What do you consider to be the three most important features of a numeration system? Why?*
- *Verification is a big mathematical idea. What is the essence of this idea and why is it so important?*

Throughout the course the reflective writing provides a record of the student's development through time, which provides new awareness and stimulus for reflection. This record also allows the teacher to enter into a dialogue with the student by responding to, challenging, and encouraging reflectiveness.

Concept Maps

Another vehicle which promotes reflectiveness is the construction of concept maps. A concept map is a representation and elaboration of a concept through the interconnectedness of this concept with other mathematical topics, objects and ideas. For example, using the word "fraction" as the original concept, a web of meaning could be developed that would illustrate one person's understanding of the concept of fraction. Other words or phrases would be linked with the original concept to demonstrate the relationship among the ideas.

Reflectiveness plays an essential role in T104. Without an awareness of oneself as a learner and doer of mathematics (one's mathematical strengths and weaknesses and one's beliefs and attitudes concerning mathematics), there is no grounds for comparing where one is mathematically and the destination to which one might aspire.

Assessment: Emphases and Practices

The various sources of data for grading in T104 are explained in the student packet on page I1 and I2. These include: class participation, tests and quizzes, homework, reflective writing, and group projects. From the confluence of these data sources, the T104 instructor assigns each student a grade that indicates the extent to which he or she has reached the goal of the course, namely, *to develop good understanding of key mathematical ideas and to be able to communicate these ideas clearly and efficiently to others.*

Yet instructors should not think of assessment just in terms of assigning grades. One just as important reason for assessment is to help the instructor build an accurate mental picture of the understandings held by students and to enable him or her to adjust instruction accordingly. Thus, assessment is an ongoing process in the T104 classroom. Whenever the instructor is circulating throughout the room during small group work, he or she should be assessing the progress of the various groups, trying to get a picture of student understandings, and making mental plans so that the wrap-up discussion or the next day's lesson may be orchestrated to help students deepen their understandings. Another source of assessment data for the instructor is the students' reflective writing. By reading students' writing an instructor gets additional insight into which topics have been understood and which need more attention or a more focused discussion.

Still another reason for assessment is to indicate to students what is considered important. Because being able to communicate about mathematics is an important goal of T104, instructors must allow sufficient time for students to talk about mathematics and to write about it. And they must offer thoughtful reactions to students' communicative efforts, so that the students can see that their efforts to explain their ideas are valued.

Teacher's and Students' Roles in T104

Students at the college level have experienced years of training in which the teacher was the authority and the teacher's word was as close to the *Truth* as was possible. As a result, many of

our college students expect the teachers to tell them what to learn and how to learn it. That is, they have not developed autonomous learning behaviors that we want them to help their future students develop.

Teacher's Role

The role of the teacher in a course such as this one is not easy. On one hand, a teacher who gives detailed directions may be sending the message that students are to be dependent on the teacher for all knowledge. But a teacher who gives almost no direction increases the possibility of student frustrations rising to a debilitating level. The teacher needs to adopt a balance, sensing when to intervene and redirect student exploration and when to allow the student to stumble along. This is not an easy task. The teacher's role is not that of an authority but more that of a guide. The teacher chooses which problems and activities to use as a means for introducing material and guides the discussion of these problems, but the teacher does not pronounce solutions. This is somewhat contrary to what has been considered appropriate teacher behavior in the past.

In this class, a heavy reliance on cooperative learning removes the teacher as the authority figure and minimizes the possibility of students blindly emulating the teacher's modelled techniques and solutions. Students are forced into helping each other develop their own deeper understanding of mathematical principles and mathematical autonomous learning behaviors.

The overarching responsibility of the teacher is to establish a mathematical community in the classroom where everyone's thinking is respected and in which reasoning and discussing mathematical ideas and meanings is the norm. Within this community, the teacher's insightful questioning can play an important role in stimulating student thinking so that there are opportunities for students to examine and question their beliefs about mathematics, to have their misconceptions challenged, and to seek clarifications, strategies, and verifications without direct teacher intervention.

Students' Role

The traditional mathematics classroom is teacher-centered. This is illustrated by the following scenario which describes a typical mathematics class period. First, answers are checked for the previous day's assignment. The more difficult porblems are worked by the teacher or students at the board. A brief explanation is given of new material and the problems assigned for the next day. The remainder of class is devoted to working on homework while the teacher is available for answering questions. This type of instruction implies that there is a specific mathematical knowledge that the teacher has and through modeling what the teacher does and through passive absorption, students will acquire this knowledge.

Educational research findings, however, indicate that learning occurs as students actively assimilate new information and construct their own meanings. Students engage in a great deal of invention in learning mathematics and impose their own interpretation on their experiences.

Knowledge is an understanding that is created through active learning. Consequently, each student's knowledge of mathematics is uniquely personal. This means that mathematics classrooms should be more student-centered. To understand what they learn, students must enact verbs that permeate all classwork: examine, represent, hypothesize, solve, communicate, prove, etc. This happens most readily when students take charge of their own learning.

In the T104 classroom, the student role must be an involved, active one. Students are expected to listen to, respond to, and question the teacher and one another. They should also be reflective thinkers to become aware of how they themselves learn mathematics. Validity of particular representations and solutions should be determined by mathematical evidence and argument, whether working in large or small groups, and not by seeking the "right" answer from the teacher. It is probably inevitable, then, that some student frustration will arise in this environment because of prior student beliefs about the nature of mathematics and expections about the proper role of the teacher. It is ultimately the task of the student to try to make sense of his/her mathematical experiences.

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PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS: A PROBLEM-SOLVING APPROACH

CONTENT COMPONENT
PUBLICATIONS AND PRESENTATIONS

Publications Related to the Content Component

Kroll, Diana Lambdin, Masingila, Joanna O., & Mau, Sue Tinsley (1992). Cooperative problem solving: But what about grading? *Arithmetic Teacher,* 39(6), 17-23..

Kroll, Diana Lambdin, Masingila, Joanna O., & Mau, Sue Tinsley (in press). Grading cooperative problem solving: Tips for teachers. *Mathematics Teacher.*

Lester, Frank K., Kroll, Diana Lambdin, Masingila, Joanna O. Mau, Sue Tinsely, Raymond, Anne M., & Santos, Vânia Maria Pereira dos. *Learning how to teach via problem solving.* Manuscript under review as a chapter for the 1994 NCTM yearbook on professional development of teachers.

Masingila, Joanna O., & Egger Moellwald, Francisco. *Using Polya to foster a classroom environment for real-world problem solving.* Manuscript submitted for publication, 1991.

Presentations Related to the Content Component (in order by date, from most recent)

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PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS:
A PROBLEM-SOLVING APPROACH
CONTENT COMPONENT

Description of Products Related to T104

Note: Products A and B below (total about 370 pages) are available in hard copy (for the cost of photocopying and optional hole punching or binding plus $2 plus postage) by contacting any *Kinkos* copy center in the nation and telling them that the books are on file in the Bloomington, Indiana *Kinkos*, 113 N. Dunn St., 812 339-3773. *Kinkos* is a nation-wide photocopy chain usually listed in the yellow pages of the phone book under "printers."

Product A: T104 Student Materials

Product A is the materials that the students purchase for use in the T104 course. Included in this packet are several sections: (1) Introduction, (2) Getting Started unit, (3) Numeration unit, (4) Operations unit, (5) Geometry/ Measurement unit, (6) Number Theory unit, and (7) Rational Numbers unit. Each section has a code for the numbering system. The codes are as follows: Introduction -- **I**, Getting Started unit -- **GS**, Numeration unit -- **NM**, Operations unit -- **OP**, Geometry/Measurement unit -- **G/M**, Number Theory unit -- **NT**, Rational Numbers unit -- **RN**.

The Introduction section has information about the course: purpose and organization of the notebook, information about assessment and grading, information about cooperative work, Course Schedule, Materials Schedule and Homework Schedule, Problem-Solving Tips sheet, and Notebook Grading Sheets. The Introduction pages are simply numbered in numerical order. The six units contain the pages to be used for class work and homework. Each day's work is assigned an integer. The pages are then numbered, in numerical order, following a decimal point. For example, the second day's activities in the Operations unit are identified by OP 2. The student pages are numbered OP 2.1, 2.2, etc.

The Course Schedule is a guide for the day-by-day activities of the course. An instructor may deviate from the schedule to fit the needs of a class. On the Material Schedule, those items designated with a "T" are the responsibility of the teacher. Those items designated with an "S" are materials that the students are responsible for bringing to class for that particular day. On the Homework Schedule, the assignments designated with an "A" are to be done before the next class period. Those designated with a "B" should be finished before the end of the unit. The Notebook Grading Sheets will be used by the instructor when grading the notebooks. Each student should be sure a blank one of these sheets is included with the notebook materials each time they are turned in to be graded.

Product B: T104 Instructor Materials

The instructor's manual contains a number of sections: (1) Teacher Introduction, (2) Getting Started unit teacher notes, (3) Numeration unit teacher notes, (4) Operations unit teacher notes, (5) Geometry/Measurement unit teacher notes, (6) Number Theory unit teacher notes, (7) Rational Numbers unit teacher notes, (8) Tests, and (9) Projects. Each unit has a code for the numbering system. The codes are as follows: Teacher Introduction -- **TI**, Getting Started Unit -- **GS**, Numeration Unit -- **NM**, Operations Unit -- **OP**, Geometry/Measurement Unit -- **G/M**, Number Theory Unit -- **NT**, Rational Number Unit -- **RN**, Tests -- **T**, and Projects -- **P**. For the six content units, the teacher notes are numbered with an integer in numerical order. The student pages for each activity are then numbered, in numerical order, following a decimal point. For example, the second activity in the Operations Unit would have the teachers' notes identified as OP 2 and the student pages identified as OP 2.1, OP 2.2, etc.

The Teacher Introduction section contains information about the goals and philosophy of T104, problem solving, the teacher's and students' roles in T104, cooperative learning, assessment, engaging the students in reflectiveness, teachers' notes, need for and use of a textbook, videotape viewing notes, tests, projects, text for the instructor orientation videotape, course syllabus, Information Sheet, and a Notebook Grading Matrix. Copies of the syllabus and information sheet are needed for each student on the first day of class. The notebook grading matrix is a suggested method of organizing the grading of the notebooks.

The teacher notes accompany each day's activities. These provide a list of the big ideas and mathematics content in these activities, a list of the needed materials, a suggested timetable, homework, and a discussion of the activities and problem solutions. Under **materials**, those items designated with a "T," are materials that the instructor needs to take for that day's activities. Those items designated with an "S" are the responsibility of the student. Under **timetable**, some time is devoted every day to discussion of previous homework. Obviously, this time can be used as needed. Under **homework**, the assignments designated with an "A" are to be done by the students before the next class period. Those designated with a "B" should be finished before the end of the unit.

Each day's teacher notes has the big ideas for that activity listed at the top of the page. In the **activity discussion** section these big ideas are italicized so that they will be brought to the instructor's attention and he or she can bring them out during the discussion time. When preparing for a lesson, it may be helpful for the instructor to work the activities first before reading the teacher's notes in order to get a better feel for what the students will encounter when working on the activities.

The tests section in this notebook provides (1) detailed instruction on the testing format for T104, (2) the group and individual follow-up problems for the group parts of the tests, (3) a scoring scale for use on the group problems, (4) answers to the group and individual follow-up problems and some sample group solutions with how they were graded, (5) sample test items for the individual parts of the tests, and (6) answers to these sample test items.

The out-of-class group projects in T104 give students the opportunity to work together as a group to explore mathematics in ways other than in traditional mathematics content (and in ways other than in the T104 content). The projects section in this notebook provides (1) directions for getting the projects underway, (2) guidelines for creating new projects, (3) descriptions of the 14 current projects, and (4) directions for doing the projects.

Product C: T104 Supplemental Materials

Three videotapes have been developed by the Mathematics Education Development Center for use with the T104 course. The first tape is a 30 minute orientation tape for instructors. This tape provides some examples of activities that illustrate the role of the T104 instructor during class time. The narration and video clips concern the activities of (1) introducing an activity, (2) commenting to small groups while they work, (3) leading class discussions, and (4) wrapping up after small group work.

The second tape has two 15 minute segments that show a group of students involved in solving problems that are worked by T104 students during the first week of class. A T104 instructor selects one of the segments (either the video clip showing some past T104 students solving the "Cereal Boxes" and "Patio Tiles" problems or the clip showing some students solving the "Weird Tables" problem) to show in class. While they are watching the tape they are to note what the problem solvers do to understand the problem, develop a plan, implement the plan and evaluate the solution. These categories are provided on a Problem-Solving Observation Guide that the students fill out while watching the tape. This videotape and the follow-up discussion provide the students with the opportunity to watch other students solve problems and to see the strategies they use. Following the class discussion the students work on a reflective writing assignment discussing the problem solving they observed.

Another opportunity to see someone else solve a problem is in the second week of class when the students watch a video clip of an "expert" problem solver modelling the process of problem solving. There are three different 30 minute clips showing the solution of two different problems. The first two clips shows Jo (first clip) and Frank (second clip) solving the Valentine's Day problem. The third clip has Diana solving the Hefty Hippos problem. The T104 instructor selects one of the video clips to be shown in his or her class after the students have worked a simpler version of the modelled problem. During the modelling of the problem solving the students use a

Problem-Solving Observation Guide to make notes of their observations. In the discussion following the viewing of the tape the instructor facilitates a discussion of the problem solving modelling and brings out the ideas that (1) understanding comes in stages (sometimes a problem has to be experienced first before it is fully understood), (2) it often takes time to solve a problem, (3) it is important to be systematic, and (4) there are different ways to represent a problem.